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Environmental Stochasticity, Cournot Competition and the Prisoner's Dilemma

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The production of natural resource-based commodities is frequently affected by environmental uncertainty and the strategic response of producers to uncertainty. We ask when uncertainty induces cooperation. Using a model of Cournot competition under environmentally induced price uncertainty, we consider the conditions under which cooperation is favored over defection. We find that the probability of cooperation depends on the length of the time period over which production levels are adjusted to price shocks. When uncertainty is low, the probability of cooperation is monotonically increasing in the length of this adjustment period. When uncertainty is high, the probability of cooperation initially increases as the adjustment period lengthens, but beyond some threshold starts to fall. In this case, the expected outcome is defection. We consider the broader implications of these findings.

Keywords: Cournot competition; price stochasticity; Itô's lemma; Wiener process.

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1. Introduction

The prices of goods or services for which either supply or demand depends on environmental conditions — such as agriculture, aquaculture, forestry, hydroelectric or wind power — are typically sensitive to environmental fluctuations. Oil prices, for example, are sensitive to a wide range of shocks, from political disturbance to

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extreme weather conditions [Kearney, 2016; Deloitte Center for Energy Solutions, 2015; EY, 2015]. For commodities like fresh water, the cost of access may fluctuate with environmental variation [Mansur and Olmstead, 2012]. Where there are a small number of producers, environmentally induced price uncertainty has long been recognized to induce a strategic response [Klemperer and Meyer, 1986].

In this paper, we explore the strategic response of firms engaged in Cournot competition to environmentally induced price uncertainty. We consider the case of a small number of producers, a small number of landowners in a watershed for example, engaged in the production of a homogeneous good. Producers are held to cooperate if they jointly produce the good at the monopoly level, and to defect if they separately produce the good at the duopoly level. The prisoner's dilemma game is an example of such competition [Gibbons, 1992].

We model the prisoner's dilemma as a "discrete" one stage Cournot competition, exploiting the fact that the Cournot–Nash equilibrium of a static duopoly with homogeneous goods, and linear price and cost functions, is an asymptotically stable steady state of the corresponding dynamic Cournot competition — albeit one in which naive expectations inform adjustment towards the best response [Bischi *et al.*, 2009; Puu, 2018; Shone, 2002; Slade, 1994; Theocharis, 1960].

We are interested both in the effect of environmentally induced price uncertainty on production decisions, and the way this varies with the time allowed for decision-makers to adjust. We refer to the latter as the adjustment period. It is analogous to the planning horizon in a dynamic analysis. The responsiveness of firms to a price change is generally more elastic in the 'long run' than in the 'short run'. This is because capital stocks are able to adjust in the long run, while in the short run they are not. Similarly, we expect strategic responses to price uncertainty that involve larger adjustments to take more time than responses that involve smaller adjustments. We ask whether the probability of a prisoner's dilemma is influenced by the length of the adjustment period.

A limited number of studies inquire into the effect of price uncertainty on production decisions under oligopoly. Pindyck [1981], for example, considered the effect of price stochasticity on resource extraction under monopoly.* Some studies apply static Cournot models that assume particular statistical distributions, and compare the profits of sequential games (e.g., Deo and Corbett [2009]). In a few cases, (e.g., Dockner *et al.* [2000]), the dynamics of state variables are characterized as Brownian motion. Itô's lemma is then used to find the stochastic evolution of the state variables.†

*Pindyck assumed that prices grow exponentially; i.e., that $dp = \alpha p dt + \sigma p dz$. This assumption is certainly consistent with the expectations of the Hotelling model [Hotelling, 1931], but not with the realized evolution of energy prices [Askari and Krichene, 2008; Borovkova and Schmeck, 2017; Deng, 2000; Ghasemi *et al.*, 2007; Jobling and Jamasb, 2017; Kanamura, 2009; Yang *et al.*, 2002].

†A possible issue with this approach is that in the standard Wiener process, the standard deviation of a state variable (random variable because the state variable is assumed to be stochastic) is proportional to the square root of time and so increases over time. In many cases, however,

Youn and Tremblay [2015], for example, model management errors as a Wiener process, and use Itô's lemma to find the expected value of each firm's output. Others have treated the size of the market as a random variable following the same Wiener process [Fujita, 2016]. The deterministic growth rate of each firm's profit in this case is treated as exponential; i.e., that $d\pi_i/\pi_i = \mu dt + \sigma dz$ where π_i is the profit of firm i and z is a Wiener process. This approach has parallels in the well-known Black–Scholes model of asset values, in which the change of the value of an asset is modeled as $dx(t)/x(t) = \mu dt + \sigma dz$. This general form has been frequently used to explore asset prices, with prices increasing in the long-run growth prospects of the economy and decreasing in consumption volatility [Bansal and Yaron, 2004; Yasue, 2003].

Another focus of research has been the choice of strategic variable: whether firms responded Cournot or Bertrand. Klemperer and Meyer [1986], for example, considered the effect of uncertainty on the Nash equilibria of a one-stage game between duopolists producing differentiated products, where the choice of prices or quantities was determined endogenously. They found that where the marginal cost curve was steep, quantity was preferred to price — primarily because of the stability of quantity at the Cournot–Nash equilibrium relative to the Bertrand–Nash equilibrium.

In what follows we consider whether uncertainty induces cooperation or defection between duopolists engaged in Cournot competition. We model price uncertainty as the sum of a deterministic time trend, and a stochastic term that captures the effect of environmental shocks on the responsiveness of prices to a variation in output. Since the shocks are exogenous, producers cannot influence their frequency or intensity. In oil markets, this might include geopolitical disturbances [Jobling and Jamasb, 2017], as well as environmentally driven demand shocks [Askari and Krichene, 2008; Yang *et al.*, 2002]. In electricity markets, it would mainly reflect environmentally driven demand shocks [Borovkova and Schmeck, 2017]. Potential sources of stochasticity include variable efficiencies of the plant used for generation at any particular moment in time [Bunn and Karakatsani, 2003], and the inability to smooth demand shocks using inventories [Deng, 2000; Li and Flynn, 2004]. Since the amount of uncertainty confronting firms is a function of the time taken to converge on the Cournot–Nash equilibrium, our results are expressed in terms of the length of the adjustment period. We find that the whether firms cooperate or defect as the length of the adjustment period/uncertainty increases depends on the impact of uncertainty on prices.

uncertainty falls over time. The extent of new mineral resources, for example, is most uncertain before the ore body has been explored. Uncertainty declines as the extent and characteristics of the ore body are revealed. So, if stochasticity in mine prices were driven by the state of knowledge of the ore body, we would expect the variability of prices to decline as information of the ore body accumulated. If price stochasticity was increasing in such cases, it would be due to other factors. It follows that use of Itô's lemma to describe the evolution of the state variables might not be appropriate in this case.

2. Discrete Cournot Competition

Our treatment of discrete Cournot competition draws on a model that first appeared in Gibbons [1992]. We assume that there are two producers of a homogeneous product who engage in a one stage game over an interval, T. The interval defines the period over which producers are able to adjust output levels to the Cournot–Nash equilibrium. It is straightforward to show that the Cournot–Nash equilibrium in this setting coincides with the steady state of a dynamic Cournot competition by the partial adjustment method explained in Bischi *et al.* [2009].

We suppose that there are two producers, each of whom is assumed to maximize a profit function of the general form:

$$\pi_i = p(\Sigma q_j)q_i - cq_i, \tag{1}$$

in which $p(\Sigma q_j)$ denotes the price of the product, $q_i > 0$ denotes output of producer i, and c denotes a constant cost per unit of output. Price is assumed to be decreasing in output, the price function taking the specific form:

$$p = p_c - k(q_1 + q_2), \tag{2}$$

where p_c denotes the choke price and k is the sensitivity of price to total output of the product.[‡] We assume that $p_c > c > 0$.

In the discrete production full information case, cooperation between producers implies that they equally share the monopoly level of output. Defection implies that they produce at the duopoly level. We denote the strategy generating monopoly output by M and the strategy generating duopoly output by D. Since monopoly output is $(p_c - c)/2k$ and monopoly profit is $(p_c - c)^2/4k$, each producer supplies $(p_c - c)/4k$ at a profit of $(p_c - c)^2/8k$ under strategy M. Under strategy D, each producer supplies $(p_c - c)/3k$ at a profit of $(p_c - c)^2/9k$. If producer 1 adopts strategy M and producer 2 adopts strategy D, profits are $5(p_c - c)^2/48k$ for producer 1 and $5(p_c - c)^2/36k$ for producer 2. The results are summarized in Table 1.

It is transparent that at the Nash equilibrium, both producers would select D although each could get a higher profit if both had selected M. We show, however, that this result is sensitive both to the length of the adjustment period and to the amount of uncertainty in the system. It is possible that both producers may choose cooperation over defection.

[‡]While price is frequently treated as nonlinear in output in Cournot competition [Canton *et al.*, 2008; Dragone *et al.*, 2012, 2013; Funaki and Yamato, 1999; Kennedy, 1994; Lee, 1999; Levin, 1985; Loury, 1986; Okuguchi, 2004; Requate, 1993; Simpson, 1995], in almost all cases, it is considered to be a C^∞ function of nonnegative real numbers so that linear approximations are possible. Since we are not concerned with the global behavior of the price or profit functions, the assumption that price is a linear function of output is reasonable. Similarly, while marginal cost may reasonably be thought to be increasing in output, preliminary analysis suggests that our results hold whether marginal costs are constant or increasing in output. Since nonlinear cost functions are more cumbersome, we use cq_i for the cost function.

Table 1. Finite Cournot Model (Producer 1's profit, Producer 2's profit).

Producer 1 \ Producer 2	M	D
M	$\left(\frac{(p_c-c)^2}{8k}, \frac{(p_c-c)^2}{8k}\right)$	$\left(\frac{5(p_c-c)^2}{48k}, \frac{5(p_c-c)^2}{36k}\right)$
D	$\left(\frac{5(p_c-c)^2}{36k}, \frac{5(p_c-c)^2}{48k}\right)$	$\left(\frac{(p_c-c)^2}{9k}, \frac{(p_c-c)^2}{9k}\right)$

3. Prisoner's Dilemma and Itô's Lemma

To see how profit is affected by uncertainty about parameter k, $dk = adt + bdw$, we use Itô's lemma.

Lemma 1 ([Itô, 2011]). *Suppose $y = F(k)$ and $dk = adt + bdw$ where w is a Wiener process. Then*

$$dy = \left(\frac{dF}{dk} \cdot a + \frac{1}{2}\frac{d^2F}{dk^2}b^2\right)dt + \frac{dF}{dk} \cdot bdw. \tag{3}$$

Since w is a Wiener process, $dw^2 = dt$ holds: uncertainty is increasing in time. To solve the problem we exploit the Euler–Maruyama method for generating approximate numerical solutions for stochastic differential equations. Using this method, we can identify the approximation, Δy as follows:

$$\Delta y = \left(\frac{dF}{dk} \cdot a + \frac{1}{2}\frac{d^2F}{dk^2}b^2\right)\Delta t + \frac{dF}{dk} \cdot b\sqrt{\Delta t} \cdot \varepsilon, \tag{4}$$

in which ε is a random variable of the standard normal distribution (i.e., $\mu = 0$ and $\sigma = 1$) and Δy is the change of value in y in response to a discrete time step Δt [Bayram *et al.*, 2018; Gherman and Borda, 2011; Kloeden and Platen, 2013]. In the Euler–Maruyama method, $\Delta t = T/N$, where N is the number of subintervals in T. Since we treat the adjustment as a one stage game, we suppose that $\Delta t = T/N = T$. What drives our choice of the Euler–Maruyama method is the fact that output adjustments in real economies are inherently discrete. Moreover, larger adjustments generally take more time than smaller adjustments, particularly if they involve changes in capital stocks. Indeed, this is precisely the distinction between the 'short' and 'long' run in economics.

First, compare π_1 at $(M,\ M)$ and $(D,\ M)$ (producer 1's strategy, producer 2's strategy). At (M, M), $\pi_1 = (p_c - c)^2/8k$, $d\pi_1/dk = -(p_c - c)^2/8k^2$ and $d^2\pi_1/dk^2 = (p_c - c)^2/4k^3$. As a result,

$$\Delta\pi_1^{(M,M)} = \left(\frac{-(p_c-c)^2}{8k^2}a + \frac{(p_c-c)^2}{8k^3}b^2\right)\Delta t + \frac{-b(p_c-c)^2}{8k^2} \cdot \varepsilon\sqrt{\Delta t}. \tag{5}$$

At (D, M), $\pi_1 = 5(p_c - c)^2/36k$, $d\pi_1/dk = -5(p_c - c)^2/36k^2$ and $d^2\pi_1/dk^2 = 5(p_c - c)^2/18k^3$. As a result,

$$\Delta\pi_1^{(D,M)} = \left(\frac{-5(p_c-c)^2}{36k^2}a + \frac{5(p_c-c)^2}{36k^3}b^2\right)\Delta t + \frac{-5b(p_c-c)^2}{36k^2} \cdot \varepsilon\sqrt{\Delta t}. \tag{6}$$

Hence, producer 1's profit at (M, M) is greater than at (D, M) if and only if

$$\pi_1^{(M,M)} + \Delta\pi_1^{(M,M)} > \pi_1^{(D,M)} + \Delta\pi_1^{(D,M)}. \tag{7}$$

i.e.,

$$\frac{(p_c - c)^2}{8k} + \left(\frac{-(p_c - c)^2}{8k^2}a + \frac{(p_c - c)^2}{8k^3}b^2\right)\Delta t + \frac{-b(p_c - c)^2}{8k^2} \cdot \varepsilon\sqrt{\Delta t}$$
$$> \frac{5(p_c - c)^2}{36k} + \left(\frac{-5(p_c - c)^2}{36k^2}a + \frac{5(p_c - c)^2}{36k^3}b^2\right)\Delta t + \frac{-5b(p_c - c)^2}{36k^2} \cdot \varepsilon\sqrt{\Delta t}. \tag{8}$$

After simplification, this reduces to

$$\varepsilon > \frac{k^2 + (b^2 - ak)\Delta t}{bk\sqrt{\Delta t}}. \tag{9}$$

That is, if producer 2 selects cooperation, the probability that producer 1 will be expected to choose cooperation over defection is more than 50% if and only if the RHS of (9) is negative.

Now, compare π_1 at (M, D) and (D, D). At (M, D), $\pi_1 = 5(p_c - c)^2/48k$, $d\pi_1/dk = -5(p_c - c)^2/48k^2$ and $d^2\pi_1/dk^2 = 10(p_c - c)^2/48k^3$. As a result,

$$\Delta\pi_1^{(M,D)} = \left(\frac{-5(P_c - c)^2}{48k^2}a + \frac{5(_c - c)^2}{48k^3}b^2\right)\Delta t + \frac{-5b(_c - c)^2}{48k^2} \cdot \varepsilon\sqrt{\Delta t}. \tag{10}$$

At (D, D), $\pi_1 = (P_c - c)^2/9k$, $d\pi_1/dk = -(p_c - c)^2/9k^2$ and $d^2\pi_1/dk^2 = 2(_c - c)^2/9k^3$. As a result,

$$\Delta\pi_1^{(D,D)} = \left(\frac{-(p_c - c)^2}{9k^2}a + \frac{(p_c - c)^2}{9k^3}b^2\right)\Delta t + \frac{-b(p_c - c)^2}{9k^2} \cdot \varepsilon\sqrt{\Delta t}. \tag{11}$$

Hence, producer 1's profit at (M, D) is greater than at (D, D) if and only if

$$\pi_1^{(M,D)} + \Delta\pi_1^{(M,D)} > \pi_1^{(D,D)} + \Delta\pi_1^{(D,D)}. \tag{12}$$

i.e.,

$$\frac{5(p_c - c)^2}{48k} + \left(\frac{-5(p_c - c)^2}{48k^2}a + \frac{5(p_c - c)^2}{48k^3}b^2\right)\Delta t + \frac{-5b(p_c - c)^2}{48k^2} \cdot \varepsilon\sqrt{\Delta t}$$
$$> \frac{(p_c - c)^2}{9k} + \left(\frac{-(p_c - c)^2}{9k^2}a + \frac{(p_c - c)^2}{9k^3}b^2\right)\Delta t + \frac{-b(p_c - c)^2}{9k^2} \cdot \varepsilon\sqrt{\Delta t} \tag{13}$$

Note that this reduces, after simplification, to exactly the same inequality given in (9), i.e.,

$$\varepsilon > \frac{k^2 + (b^2 - ak)\Delta t}{bk\sqrt{\Delta t}}. \tag{14}$$

If producer 2 selects defection, the probability that producer 1 will again be expected to choose cooperation over defection is more than 50% if and only if the RHS of (14) is negative. While particular realizations of $\varepsilon > 0$ may lead to cooperation, since its expected value is zero, cooperation would not be expected unless the RHS of (9) and (14) are negative. The probability that producer 1 chooses strategy M over D accordingly depends on the slope of the price function, the degree of price stochasticity, and the length of the adjustment period.

To see how the probability of cooperation varies with the period over which price stochasticity is evaluated, consider the behavior of the right-hand side of (9) and (14) as Δt increases. Let

$$f(\Delta t) = \frac{k}{b\sqrt{\Delta t}} + \frac{(b^2 - ak)\Delta t}{bk\sqrt{\Delta t}}, \tag{15}$$

and consider how $f(\Delta t)$ changes as Δt increases. Differentiation yields

$$f'(\Delta t) = -\frac{k}{2b}(\Delta t)^{-1.5} + \frac{b^2 - ak}{2bk}(\Delta t)^{-0.5}. \tag{16}$$

Obviously, $f'(\Delta t) < 0$ if $b^2 - ak \leq 0$ for all $\Delta t > 0$. It follows that the probability of cooperation increases with the length of the adjustment period for all parameter values consistent with $b^2 - ak \leq 0$.

On the other hand, when $b^2 - ak > 0$, whether the probability that cooperation increases or decreases depends on the length of the adjustment period. There is a critical Δt such that $f'(\Delta t) = 0$; i.e., $f'(\Delta t) = 0$ when $\Delta t = \frac{k^2}{b^2 - ak}$. For longer adjustment periods, $f'(\Delta t) > 0$ implying that the probability that both producers choose M (cooperate) decreases. For shorter adjustment periods, $\Delta t < \frac{k^2}{b^2 - ak}$, $f'(\Delta t) < 0$ implying that the probability that both producers choose M increases.

To illustrate, consider a numerical example. First, take the case where $b^2 - ak \leq 0$; i.e., let $b = 0.1$, $a = k = 1$. Then

$$f(\Delta t) = \frac{10}{\sqrt{\Delta t}} - 9.9\sqrt{\Delta t}. \tag{17}$$

When $\Delta t = 1, f(1) = 0.1$ so that

$$P(\varepsilon > 0.1) = 0.46. \tag{18}$$

The probability of the emergence of cooperation is 0.46.

Next, consider the case where $b^2 - ak > 0$; i.e., let $a = k = 1$ and $b = 2$, then

$$f(\Delta t) = \frac{1 + 3\Delta t}{2\sqrt{\Delta t}}. \tag{19}$$

When $\Delta t = 1$, $f(1) = 2$ so that

$$P(\varepsilon > 2) = 0.023. \tag{20}$$

Hence, the probability of the emergence of cooperation is 0.023.

In addition, when $b^2 - ak > 0$, there is a critical time such that $f'(\Delta t) = 0$. If $a = k = 1$ and $b = 2$, the critical adjustment period is $\Delta t = 1/3$. Figure 1 graphs the cases of different values of b as well as the two cases above.

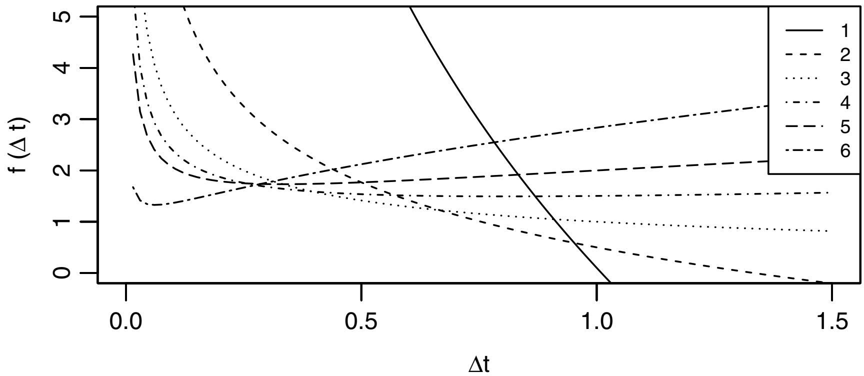

Fig. 1. The graph of $f(\Delta t)$ depending upon the values of $b^2 - ak$ where $a = k = 1$ 1: $b = 0.1$, 2: $b = 0.5$, 3: $b = 1$, 4: $b = 1.5$, 5: $b = 2$, 6: $b = 3$. In cases 1–3, $f(\Delta t)$ is strictly decreasing implying that the probability that cooperation emerges is strictly increasing in the length of the adjustment period. In cases 4–6, $f(\Delta t)$ is increasing beyond some point, implying that the probability that cooperation emerges is decreasing in the length of the adjustment period after that point.

Last, we make a remark on the relation between our result and the folk theorem. Consider the following numerical example: $p_c = 9, c = 1$ and $k = 1$. This yields the prisoner's dilemma described in Table 2.

Now, the folk theorem states that if the game is repeated infinitely many times, and if the discount factor is close to 1, then any combinations of payoffs in the shaded region in Fig. 2 can be achieved as the average payoff in a subgame perfect Nash equilibrium of the repeated game [Gibbons, 1992].

Table 2. Finite Cournot Model (Producer 1's profit, Producer 2's profit) $p_c = 9, c = 1$ & $k = 1$.

Producer 1\Producer 2	M	D
M	$(8, 8)$	$(6.67, 8.89)$
D	$(8.89, 6.67)$	$(7.11, 7.11)$

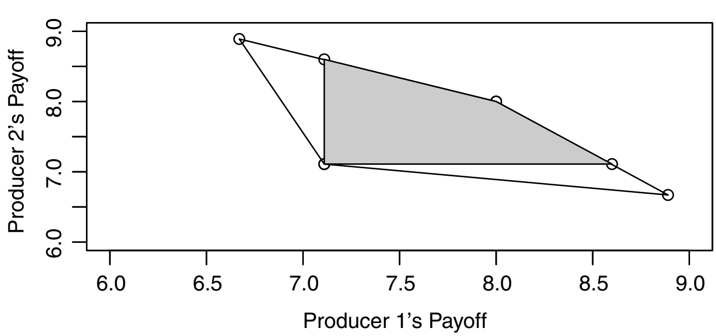

Fig. 2. Any combinations of payoffs in the shaded region can be achieved in infinitely repeated games, provided the discount factor is close to 1.

We noted earlier that in the nonrepeated one stage game discussed in this paper, if $b^2 - ak \leq 0$, the probability of the emergence of cooperation increases as the evaluation period lengthens. When the game is deterministic, $b = 0$ and a and k are positive so that $b^2 - ak \leq 0$ is satisfied. Hence, our result is not only consistent with the folk theorem, but also provides an additional proof of the potential for cooperation in deterministic prisoner's dilemma games evaluated over a long enough period. Moreover, the result is robust with respect to the addition of limited uncertainty, although the period required before cooperation is the expected outcome lengthens.

4. Discussion and Concluding Remarks

Our main result can be summarized quite simply. If there is no uncertainty and the adjustment period approaches zero, both producers always choose defection. That is, the game takes the form of a prisoner's dilemma. However, if there is uncertainty and the adjustment period is positive, there is a possibility that both producers choose cooperation; i.e., price uncertainty can lead to the selection of cooperation over defection, depending on the values of a, b and k. The optimal strategy in this case is, however, highly sensitive to the length of the adjustment period, Δt.

The three determinants of output prices, a, b and k, denote the marginal effects of time, uncertainty and competition, respectively. Of these, the time trend has an unambiguously positive effect on the probability of cooperation. The effects of uncertainty and competition are more sensitive to the specific values of both parameters. In general, the stronger the price effect of uncertainty, the less likely producers are to cooperate as the adjustment period lengthens. Although the marginal effect of a change in the length of the adjustment period is diminishing, the probability that $\varepsilon > f(\Delta t)$ is strictly increasing in t as b falls/k rises.

If $b^2 - ak \leq 0$, the probability of the emergence of cooperation increases as the adjustment period lengthens. On the other hand, if $b^2 - ak > 0$, there exists a critical length of the adjustment period at which the optimal strategy switches from cooperation to defection. For adjustment periods less than this, the probability of cooperation increases. For adjustment periods greater than this, the probability of cooperation decreases. When a, b and k satisfy $b^2 - ak > 0$, environmentally induced uncertainty increases the likelihood of cooperation, but only for adjustment periods below the critical value.

Recall that the adjustment period is the period during which producers are able to adjust output to the Cournot–Nash equilibrium. If adjustment involves changes in capital stocks, the period will be longer. If not, it will be shorter. This is analogous, for example, to the implicit assumption in a Walrasian model that the adjustment period is long enough to accommodate a tatonnement process. The significance of the length of the adjustment period in this paper is that it subsumes the time-dependent effects of variation in environmental conditions on prices. The longer the adjustment period, the greater the overall impact of such effects. While we have not specified the problem in terms of the optimization of a discounted stream of benefits over time, there are clear parallels between the adjustment period in this case and the time horizon in the net present value maximization case.

Finally, if there is no uncertainty and adjustments are not instantaneous, our result is consistent with the folk theorem. We show that in the fully deterministic case, or the case where uncertainty is low, the probability of the emergence of cooperation increases as the adjustment period lengthens. This is exactly what the folk theorem indicates when the discount factor is close to 1.

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