Strategic Behavior and the Environment, 2016, 6: 311–338

Potential Games and the Tragedy of the Commons

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ABSTRACT

The term tragedy of the commons is widely used to describe the overexploitation of open access common pool resources. Open access allows potential resource users to continue to enter the resource up to the point where rents are exhausted. The resulting level of resource use is higher than the socially optimal level. In extreme cases, unlimited entry can lead to the collapse of the resource and the communities that depend on it. In this paper we use potential games to analyze the relation between costs of entry, costs of production, and the equilibrium number of resource users in open access regimes. We find that costs of access and costs of production determine the equilibrium number of resource users. We also find a natural link between Cournot competition and the tragedy of the commons. We discuss the relation between common pool resource management regimes and cost structure and show that cost structures are sufficient to determine the number of resource users accessing the resource.

ISSN 1944-012X; DOI 10.1561/102.00000079

Keywords: Common pool resources; potential games; the tragedy of the commons; collapse of societies

JEL Codes: P14, Q15, Q32, Q56

1 Introduction

Ever since the publication of Garrett Hardin's influential paper, the causes and consequences of the tragedy of the commons have been a topic of debate among social and life scientists. The archetype of the tragedy of the commons is as follows. Suppose that, in a rural village, all villagers have access to a common parcel of land on which each is entitled to let his or her cattle graze — the commons. Because access is open, villagers have an incentive to enter the commons as long as it is profitable to them to do so. The 'tragedy' lies in the fact that each animal added to the commons reduces the graze/browse available to all other animals, yet each villager adding animals has no incentive to take the wider costs of his actions into account. The result is collective overexploitation of the resource. There are certainly examples of common pool resources that have been degraded in this way (Dasgupta, 2001; Feeny *et al.*, 1990; Libecap, 2009; McWhinnie, 2009; Ostrom *et al.*, 2002). At the same time, however, there are as many or more examples of common pool resources that have been managed sustainably (Berkes *et al.*, 1989; Dolsak and Ostrom, 2003; Ostrom, 2015; Seabright, 1993). The question this has raised in the 50 years since the appearance of Hardin's paper is when has open access led to the overexploitation of common pool resources, and when has it not?

In this paper we address this question via the factors that determine the number of common pool resource users. Common pool resources include fisheries, forests, rangelands, and water resources. They include wildlife reserves and protected areas, heritage sites, and shrines. They also include regulatory functions of ecosystems, such as storm buffering, erosion control, or pest predation. In the limit they include planetary scale resources such as the atmosphere and the oceans (Perrings, 2014). The number of people accessing each type of resource varies widely — from a mere handful to the population of the planet.

We ask when open access leads all potential users to exploit the resource, and when it does not. We do this against the background of an extensive literature on common pool resources and the tragedy of the commons: a literature that is primarily focused on the conditions that lead a finite number of entrants to limit activity levels. It has three main elements.

The first is a large number of case studies on the role of regulatory institutions in mitigating the tragedy of the commons in particular common pool resource systems — by restricting either the total number of entrants, or the level of activity of individual entrants (Abbott and Wilen, 2011; Berkes *et al.*, 1989; Dietz *et al.*, 2003; Feeny *et al.*, 1990; Ostrom, 2015; Pretty, 2003). This literature is based on more than thousand empirical studies that collectively show that far from leading inexorably to ruin, common property in natural resources most often leads to the development of mechanisms that regulate extraction to sustainable levels. The empirical literature has, in turn, stimulated a series of experimental and theoretical studies designed to ask how and why.

The second element comprises reports of the results of numerous behavioral experiments on common pool resource games. These includes experiments constructed to test the effect of a range of conditions thought to affect behavior, including entry restrictions (Walker *et al.*, 1990), allocation rules (Walker *et al.*, 2000), rules of capture and stock quota (Gardner *et al.*, 2000), strategic design (Keser and Gardner, 1999), and punishment (Casari and Plott, 2003). Experimental studies have also considered the effect on activity levels of cooperation (Mason and Phillips, 1997), altruism (Fischer *et al.*, 2004), reciprocity, inequity aversion, and conformity (Velez *et al.*, 2009), covenants (Ostrom *et al.*, 1992), and communication (Cardenas *et al.*, 2004; Ostrom, 2006). Many experimental studies reveal behaviors that differ from those one would expect from non-cooperative game theory. Participants in common pool resource experiments are, for example, frequently willing to restrict resource extraction and punish those who defect (Anderies *et al.*, 2011), although experiments involving cooperation amongst subgroups have typically shown overexploitation to be avoidable only if all participants are engaged (Bernard *et al.*, 2013).

The empirical and experimental findings have, in turn, affected theoretical work on the problem. Beginning with Hotelling's seminal study of the optimal exploitation of non-renewable resources (Hotelling, 1931), and two studies of behavior in common pool fisheries (Gordon, 1954; Schaefer, 1957), a theoretical literature on common pool resource games has developed, focused either on the effect of path strategies on Nash equilibria or on the effect of decision-rule strategies on subgame perfect Nash equilibria (Reinganum and Stokey, 1985; Van Long, 2011).

Initially conceptualized as a prisoners' dilemma (Dawes, 1980), the tragedy of the commons was for some time analyzed as a non-cooperative game, the specific game form depending on the characteristics of interest. Most concern was on the effect of open access on exploitation rates when users had more or less market power (Clark, 1976; Clark *et al.*, 1973; Clark, 2010; Dasgupta and Heal, 1979; Eswaran and Lewis, 1984; Khalatbari, 1977). As empirical research results emerged revealing the many mechanisms by which local communities regulated common pool resources (Ostrom, 2015), attention shifted to a different set of game forms. For example, it was suggested that common pool resource management in a village setting generally involves a repeated coordination game (Runge, 1984). For small numbers of resource users, management of common pool resources was frequently treated as a cooperative game subject to binding agreements (Funaki and Yamato, 1999; Uzawa, 2005). Indeed, all of the experimental studies referred to above, and many of the theoretical studies take this position.

Later studies focused on factors limiting the effort of those choosing to enter the resource. These factors include the role of uncertainty (Sandler and Sternbenz, 1990), the evolutionary role of social norms of restraint and punishment theoretic framework (Sethi and Somanathan, 1996), and the role of cooperation (Funaki and Yamato, 1999). A number of studies showed how different game structures, repetition frequencies, and player characteristics could encourage beneficial behavior (Faysse, 2005). Interestingly, less attention was paid to the incentives facing potential entrants to a common pool resource (see Dragone *et al.*, 2013; Mason and Polasky, 1997). A well-known example is the lobster fishery in Maine. Though frequently analyzed as a coordination game, it is clear that it also involved a very particular set of incentives. Acheson (2003) described the system at that time as follows:

> To go lobstering, one needs a state license, which ostensibly allows a person to fish anywhere in state waters. In reality, more is required. One also needs to gain admission to a "harbor gang" that maintains a fishing territory for the use of its members (24).

In this system, each harbor gang comprises a small group of fishers, perhaps as few as six or eight boats, controlling territories 100 square miles or less in area. There are two types of territories: nucleated and perimeter-defended. Acheson (2003) noted that entry into the harbor gangs that controlled nucleated territories was easier than entry into the perimeter-defended areas. Nevertheless, there were a range of informal "costs" associated with entering both nucleated and perimeter defended territories, and these were increasing in the number of fishers.

The reduction in the numbers of fishers imposed by the informal system has had positive effects on total productivity in the fishery. Catches that were reported to be at record-high levels at the beginning of the Century (Acheson, 2003) have continued to rise. In 2016, fishers landed more than 130 million pounds of lobster (valued at \$533 million), nearly three times the catch level in 2000 (Overton, 2017).

The central problem we wish to address is that the payoff to each resource user depends on the total number of users accessing the resource. The mechanisms involved typically differ from case to case. If the size of the population having access rights is small, the mechanisms may involve agreed rules of access. However, while the establishment and enforcement of binding agreements among a small number of resource users are reasonable (Ostrom, 2015), cooperation becomes less and less likely as the number of resource users increases (Dietz *et al.*, 2003). To approach the problem in a large number setting we treat access to common pool resources as a non-cooperative congestion game. By treating the problem as a congestion game we are also able to exploit a convenient property of congestion games — that they are isomorphic to potential games (Monderer and Shapley, 1996; Rosenthal, 1973). That is, we are able to describe the incentive that all resource users have to change their strategy in terms of a single global function, the potential function.

The pure Nash equilibrium emerges from a process in which resource users react by selecting a strategy that maximizes the benefit to them (Gourves *et al.*, 2015). This turns out to be an extremely flexible approach, allowing us to explore equilibrium numbers of resource users from one to infinity. For very small numbers we are able to exploit the fact that Cournot competition is an example of a potential game (Monderer and Shapley, 1996). That is Cournot competition and the tragedy of the commons both belong to a class of problems in which the structure of costs uniquely determines the number of resource users. While we identify the impact of the equilibrium number of users on productivity and profitability in the exploitation of the common pool resource itself, we do not consider the wider consequences of overexploitation of common pool resources beyond noting the claim that, in extreme cases, overexploitation can lead to societal collapse (Dasgupta *et al.*, 2016; Diamond, 2005).

We take exhaustion of rents to be the primary evidence for overexploitation, of common pool resources noting that at the Nash equilibrium resource users receive a share of output equal to their share of effort, and that they equate the price of output to the weighted sum of the marginal and average product of assets. As the number of resource users increases, the weight assigned to average product increases, and as the number approaches infinity, profits approach zero (Sandler *et al.*, 2003). In the standard approach to the problem (e.g., Dasgupta and Heal, 1979; McCarthy *et al.*, 2001), both relative prices and the number of resource users are fixed and treated as exogenous.

In this paper we follow these authors in treating strategic behavior as non-cooperative, but we also treat the number of resource users as endogenous. More particularly, we identify the number of resource users at the Nash equilibrium corresponding to the cost structure associated with the resource. We consider the case where the commons game is symmetric. All users produce at the same level and face the same costs. For particular cost structures we find that the equilibrium number of users may be infinite, while for other cost structures the number will be finite and decreasing in the cost of access or production.

2 Potential Commons Games

2.1 Approaches to CPR Games

The core concept applied in this paper is that of the potential game. The potential game uses a potential function to identify the Nash equilibrium. This has two advantages. First, using a potential function to find the Nash equilibrium is generally easier than using the "traditional" method of finding the Nash equilibrium (e.g., elimination of dominated strategies). Second, the Nash equilibrium found from the potential function is the one that most likely emerges when the game is played if there are several Nash equilibria in the game. The reason is that the Nash equilibria found from potential games are stochastically stable.

Our approach differs from the recent study by Dasgupta *et al.* (2016) which discusses commons games using the concept of Markov Perfect Equilibria, a refinement of subgame perfect equilibrium (Fudenberg and Tirole, 1991). Since a subgame perfect equilibrium is not necessarily evolutionarily stable (Samuelson, 1998), the set of Markov perfect equilibria may change discontinuously if payoffs are perturbed (Fudenberg and Tirole, 1991). Exploiting this property, Dasgupta *et al.* (2016) argue that, in case of the depletion of common pool resources, "[a] sudden crash in productivity, population overshoot, or decline in harvesting costs can tip an unmanaged common into ruin" (1). The problem we consider is slightly different. We are not concerned with the likelihood that some perturbation may induce the collapse of an open access resource, but the conditions in which open access makes collapse inevitable. This is partly motivated by the evidence that collapse has rarely been abrupt (Butzer, 2012). We look instead for properties of the system that lead it to collapse, albeit over much longer periods of time.

Although there are precursors to the idea of potential games in the literature on strategic behavior, it was the paper by Monderer and Shapley (1996) that formally organized ideas about potentials that had been scattered across various disciplines and structured those ideas follows. Suppose $\Gamma = (N, A, u)$ denotes a strategic form game, and that there is a finite number of players, $N = 1, \ldots, n$. A_i is a set of strategies for player $i \in N$ where $A = (A_i)_{i \in N}$, and $u_i \in \mathbf{R}$ is a payoff function for player $i \in N$. $a = (a_1, a_2, \ldots, a_n)$ and $a_{-i} = (a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$ where $a_i \in A_i$.

Definition (Monderer and Shapley, 1996)**.** $\Gamma = (N, A, u)$ is called an exact potential game if there exists a function $\Pi : A \to \mathbf{R}$ such that

$$u_i(a_i^{'}, a_{-i}) - u_i(a_i, a_{-i}) = \Pi(a_i^{'}, a_{-i}) - \Pi(a_i, a_{-i})$$

for any $i \in N$, $a_i, a_i^{'} \in A_i$ and $a \in A$.

If set A is continuous and u_i and Π are differentialable on A_i, then the differences in the payoff function and the potential function are replaced by derivatives; i.e., $\Gamma = (N, A, u)$ is an exact potential game if there exists a function $\Pi : A \to \mathbf{R}$ such that

$$\frac{\partial u_i}{\partial a_i} = \frac{\partial \Pi}{\partial a_i}$$

for any $i \in N$ and $a_i \in A_i$. Moreover, Monderer and Shapley (1996) clarify that the necessary and sufficient condition for the continuous game to have a potential function is

$$\frac{\partial^2 u_i}{\partial a_i \partial a_j} = \frac{\partial^2 u_j}{\partial a_i \partial a_j}$$

for all i and j. This condition provides a convenient criterion for testing whether any continuous game is a potential game. Together with the relationship of the first-order derivatives above, it can be used as a tool to find the potential function by taking anti-derivatives. We note that this condition is related to the fact that costs and revenues are shared by all resource users equally in symmetric games.

Potential games are particularly useful because, as the following theorem states, the Nash equilibrium found from the game in which each utility function is replaced by the potential function is identical to that of the game with the original utility functions.

Theorem (Monderer and Shapley, 1996). *Let* $\Gamma = (N, A, u)$ *be an exact potential game with a potential function* Π. *Let* $\overline{\Gamma}$ *be the game with* (N, A, Π) *in which every player's payoff function is* Π. *Then, the set of Nash equilibria of* Γ *coincides with that of* $\overline{\Gamma}$.

This theorem ensures that a single potential function can be used to find all the Nash equilibria of a game, so simplifying analysis. Moreover, for continuous exact potential games, if A_i is compact for all $i \in N$, then the game has at least one pure strategy Nash equilibrium; i.e., $a^* \in \underset{a \in A}{arg\,max}\, \Pi(a)$ is a Nash equilibrium (Foster and Young, 1990; Monderer and Shapley, 1996; Ui, 2000).

The implication is that potential games can be studied from two different perspectives. First, they can be studied within the classical framework of game theory. Second, they can be studied through optimization of the potential function (Goyal, 2012; Monderer and Shapley, 1996; Slade, 1994). In this paper, we use the optimization framework.

The Nash equilibrium is found by identifying the *argmax* of the potential function with a fixed n. We then consider the outcome when n varies.

We use the fact that Nash equilibria found from potential functions belong to a special class of equilibria — stochastically stable equilibria (Foster and Young, 1990). A state P is a stochastically stable equilibrium if, in the long run, it is nearly certain that the system lies within every small neighborhood of P as noise tends to zero. That is, the stochastically stable set is the set of states S such that, in the long run, it is nearly certain that the system lies within every open set containing S as noise tends to zero.[1]

What is significant here is that the potential function of the game attains the global maximum at the stochastically stable equilibrium (Alós-Ferrer and Netzer, 2010; Foster and Young, 1990; Goyal, 2012). There is, however, a restriction to this result. Alós-Ferrer and Netzer (2010) find that, for some exact potential games, the stochastically stable equilibrium coincides with the argmax of the potential function only if players revise their strategies based on "asynchronous learning"; i.e., exactly one player is randomly selected every period to revise his or her strategy. If revisions of the strategies are not asynchronous (e.g., every player revises his or her strategy at the same time), the realized Nash equilibrium may not maximize the potential. Specifically, we consider a common pool resource in which potential users decide sequentially whether to enter the resource. One reason why this might occur is that individuals located nearer or further from the resource may face differential costs of access.

In what follows we also make use of the fact that all potential games with a finite number of players are congestion games with the same potential function (Monderer and Shapley, 1996). The commons game is a finite potential (congestion) game; i.e., the number of players, n, is finite, albeit large. If the number of players is finite, potential

[1]This concept of equilibrium relates to the process of adaptive learning in games. A player observes the history of how other players have played against him or her in the past, and chooses a strategy for the future that is a best response to the past play of others (Gintis, 2009). Adaptive learning in games allows players to make errors when they perceive how others have behaved, given past realizations of the system. But as the game continues, players learn from these mistakes and the frequency of errors becomes lower. In this way, the stochastically stable equilibrium is attained.

games are always isomorphic to congestion games. Moreover, since Sandholm (2001) showed that if players are anonymous and identical, then continuous congestion games can be defined as the limit of atomistic congestion games (i.e., in which the number of players is finite). It follows that our potential commons games are indeed isomorphic to congestion games.

2.2 Modeling the Commons Game

We suppose that $n(0)$ potential resource users consider whether to access the commons sequentially. Each potential resource user chooses to enter the commons if and only if his/her net profit from entering is positive. This depends on the number of resource users already in the commons. If there are n resource users in the commons, we assume that all produce at the Cournot–Nash equilibrium level $q_i = q^*(n)$ for all i. This is the level at which no resource user has an incentive to change his/her level of effort. Since costs are increasing (profits are decreasing) in n, the $n + 1^{th}$ resource user enters if and only if his/her net profit from entering, $\pi_i(n + 1, q^*(n + 1))$, is positive. Furthermore, we consider $n(0)$ to be large. The number of resource users approaches an equilibrium level n^* when either no more users are entitled to enter, $n^* = n(0)$, or eligible resource users are not willing to enter because $\pi_i(n^*, q^*(n^*)) = 0$.

To determine the profit for a given resource user, we begin with a model of the commons game by Gibbons (1992) in which the number of players who actually access the resource, n, is assumed to be fixed and finite. The profit to the i^{th} resource user is given by:

$$\pi_i = v(G) \cdot q_i - cq_i$$

where $v(G)$ is the revenue per unit of output which depends on total production, $G = \sum q_i$, and c is the cost per unit of output. Specifically, Gibbons (1992) assumes that the revenue per unit of output decreases with G ($v^{'}(G) < 0$) at a diminishing rate ($v^{''}(G) < 0$). If we take the example of Hardin's grazing lands, this reflects the decline in quality of individual animals as the number of animals on the commons increases. When there are few resource users in the commons, the addition of one more resource user does little harm. When there are already many resource users in the commons, however, the addition of one more resource user significantly harms the rest (Gibbons, 1992). Note that, in this example, Gibbons (1992) assumes that each animal produces the same amount of output, and that every resource user owns the same number of animals.

Next, we modify Gibbons' model to allow costs to increase with n. Specifically, we consider two variants of the model. In both cases, production costs depend on n and there is a constant cost of access, δ. In the first variant, production cost increases linearly in output,

$$\pi_i^a = v(G) \cdot q_i - cn^\gamma q_i - \delta \tag{1}$$

where $0 \le \delta \le l$, $\gamma \ge 0$, and $n = 1, 2, \ldots, n(0)$.

In the second variant, production costs are quadratic in output,

$$\pi_i^b = v(G) \cdot q_i - cn^\gamma q_i^2 - \delta \tag{2}$$

Note that the costs of production, $cn^\gamma q_i$ and $cn^\gamma q_i^2$, are congestion costs in the sense that they increase with the number of users in the commons.

Finally, we choose a particular function, $v(G)$, that makes our game a potential game. We restrict our attention to the symmetric Nash equilibrium given by $q_1^* = q_2^* = \cdots = q_n^* = q^*$.

Specifically, we assume that $v(G) = l - G^2$ is given by the linearization of $f(G) = G^2$ around the equilibrium $q_i = q^*$.

$$\begin{aligned} f(G) &\approx f(G)\bigg|_{q^*} + \frac{\partial f}{\partial q_1}\bigg|_{q^*}(q_1 - q^*) \\ &\quad + \frac{\partial f}{\partial q_2}\bigg|_{q^*}(q_2 - q^*) + \cdots + \frac{\partial f}{\partial q_n}\bigg|_{q^*}(q_n - q^*) \\ &= n^2 q^{*2} + 2nq^*(q_1 - q^*) + 2nq^*(q_2 - q^*) + \cdots + 2nq^*(q_n - q^*) \\ &= n^2 q^{*2} + 2nq^*\left(\sum_{k=1}^{n} q_k - nq^*\right) \end{aligned}$$

Using this expression for revenue, we can then rewrite Equations (1) and (2) as follows:

$$\pi_i^a = \left\{ l - n^2 q^{*2} - 2nq^*\left(\sum_{k=1}^{n} q_k - nq^*\right)\right\} q_i - cn^\gamma q_i - \delta \tag{3}$$

and

$$\pi_i^b = \left\{ l - n^2 q^{*2} - 2nq^* \left(\sum_{k=1}^{n} q_k - nq^* \right) \right\} q_i - cn^\gamma q_i^2 - \delta \tag{4}$$

Next, to see that profit function (Equation (4)) has a potential, note that

$$\frac{\partial \pi_i^b}{\partial q_j} = -2nq^* q_i \Rightarrow \frac{\partial^2 \pi_i^b}{\partial q_i \partial q_j} = -2nq^*$$

and

$$\begin{aligned} \frac{\partial \pi_j^b}{\partial q_j} &= l - n^2 q^{*2} - 2nq^* \left(\sum_{k=1}^{n} q_k - nq^* \right) - 2nq^* q_j - 2cn^\gamma q_j \\ &\Rightarrow \frac{\partial^2 \pi_j^b}{\partial q_i \partial q_j} = -2nq^* \end{aligned}$$

Consequently,

$$\frac{\partial^2 \pi_i^b}{\partial q_i \partial q_j} = \frac{\partial^2 \pi_j^b}{\partial q_i \partial q_j}$$

so that an exact potential exists for π_i^b for a given n. A similar result holds for π_i^a. Specifically for π_i^a, the potential function is given by

$$\begin{aligned} \Pi^a &= \sum_{k=1}^{n} \left\{ l - n^2 q^{*2} - 2nq^* \left(\sum_{m=1}^{n} q_m - nq^* \right) \right\} q_k - cn^\gamma \sum_{k=1}^{n} q_k \\ &\quad + nq^* \left(\sum_{k=1}^{n} q_k \right)^2 - nq^* \sum_{k=1}^{n} q_k^2 \end{aligned} \tag{5}$$

and for π_i^b, the potential is given by

$$\begin{aligned} \Pi^b &= \sum_{k=1}^{n} \left\{ l - n^2 q^{*2} - 2nq^* \left(\sum_{m=1}^{n} q_m - nq^* \right) \right\} q_k - cn^\gamma \sum_{k=1}^{n} q_k^2 \\ &\quad + nq^* \left(\sum_{k=1}^{n} q_k \right)^2 - nq^* \sum_{k=1}^{n} q_k^2 \end{aligned} \tag{6}$$

To verify that Π is the potential, note that $d\Pi^a/dq_i = d\pi_i^a/dq_i$ and $d\Pi^b/dq_i = d\pi_i^b/dq_i$ for all i. The Nash equilibrium for a given n is

$$q^* = \begin{cases} \sqrt{\dfrac{l - cn^\gamma}{n^2 + 2n}} & \text{if } n \leq (l/c)^{1/\gamma} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

for π_i^a and

$$q^* = \frac{\sqrt{c^2 n^{2\gamma} + l(n^2 + 2n)} - cn^\gamma}{n^2 + 2n} \tag{8}$$

for π_i^b. (See Appendix A.)

3 The Equilibrium Number of Resource Users under Open Access

3.1 Case 1: Marginal Cost Constant in Output

First, we consider the case where profit is given by π_i^a (cost increases linearly in output) and there is no access cost ($\delta = 0$). Resource users have an incentive to enter the commons as long as the profit from doing so, $\pi_i^a(n, q^*(n))$, is greater than the profit from staying outside (zero). Specifically, when $\delta = 0$, $\pi_i^a(n, q^*(n)) > 0$ if and only if $q^*(n) > 0$ (Figure 1).

It follows from Equation (7) that resource users enter the commons as long as $l - cn^\gamma > 0$; i.e., as long as $n < (l/c)^{(1/\gamma)}$. Hence, the equilibrium number of resource users is given by $n^* = \min(n(0), (l/c)^{(1/\gamma)})$. Note that n^* increases as γ decreases (Figure 1). Specifically, $n^* \to n(0)$ for sufficiently low γ.

If access cost is positive, $\delta > 0$, resource users enter the commons as long as $\pi_i^a(n, q^*(n)) \geq 0$. As in the case where $\delta = 0$, $\pi_i^a(n, q^*(n)) = 0$ at some finite value $n = n^*$. Since the equilibrium number of resource users is decreasing in δ, it is strictly less than the equilibrium number of resource users when access costs are zero.

3.2 Case 2: Marginal Cost Increasing in Output

Next, we consider the case where profits are given by π_i^b (production costs are quadratic in output) and again assume that access cost is zero.

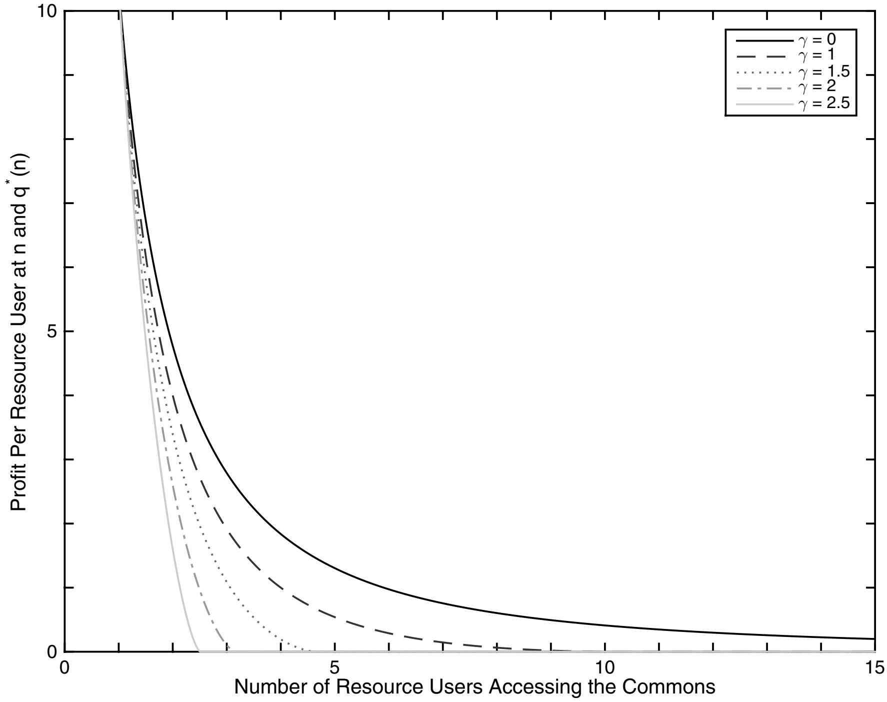

Figure 1: Profit per resource user for each n in the commons when marginal cost is constant and access cost is zero.
The curves show profits per resource user, π_i^a (Case 1 Equation (3)), as a function of the number of resource users in the commons, n, assuming they produce at $q^*(n)$ (Equation (7)) for various values of γ. Here, we assume no access cost ($\delta = 0$). The equilibrium number of resource users is given by $q^*(n^*) = 0$, where the profit curves intersect with the horizontal axis. Specifically, note that n^* decreases as γ increases. Parameters: $l = 10$, $c = 1$.

From Equation (8), it follows that $q^*(n) > 0$ for all n and γ. Hence, for all values of $\gamma \geq 0$, each resource user's profit is positive for all values of $n \geq 0$; therefore, $n^* = n(0)$. Figure 3 shows the graph of Equation (4), a linear approximation of Equation (2), for different levels of congestion costs (γ) given $\delta = 0$.

In this case, q tends to 0 as $n(0)$ approaches infinity for all values of γ, even though resource users equate marginal revenue and marginal cost.

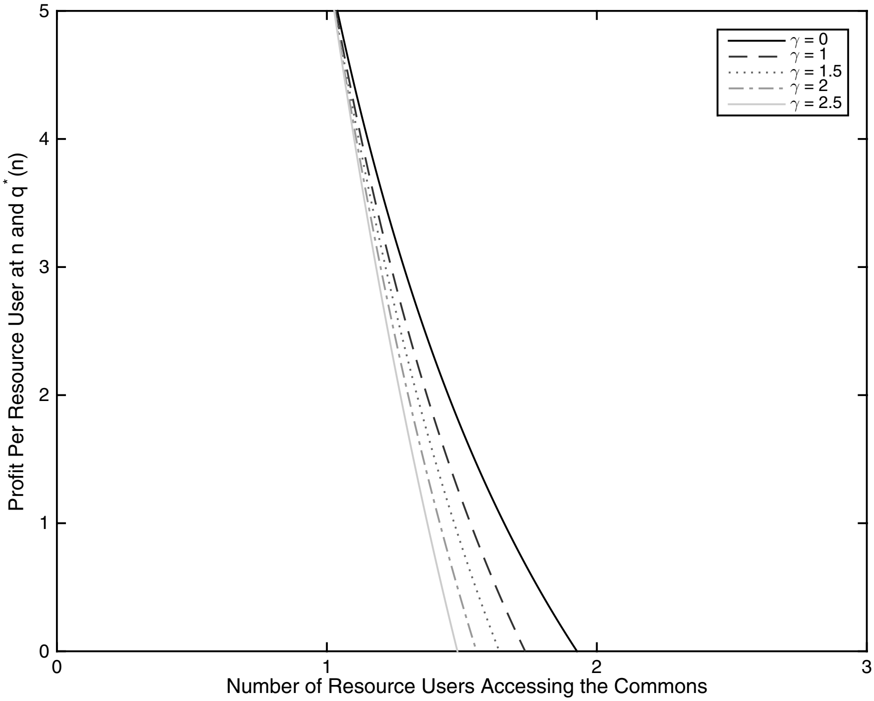

Figure 2: Profit per resource user for each n in the common when marginal cost is constant and access cost is positive.
The curves show profits per resource user, π_i^a (Case 1 Equation (3)), as a function of the number of resource users in the commons, n, assuming they produce at $q^*(n)$ (Equation (7)) for various values of γ. Here, we consider a positive access cost ($\delta = 5$). The equilibrium number of resource users is given by $\pi_i^b(n^*, q^*(n^*)) = 0$, where the profit curves intersect with the horizontal axis. Specifically, note that n^* decreases as γ increases. Parameters: $l = 10$, $c = 1$.

If production costs are quadratic in output and access costs are positive, $\delta > 0$, we find a similar impact on the equilibrium number of resource users to that described in Case 1 (Figure 4).

Since δ shifts down all profit curves, $\pi_i^b(n^*, q^*(n^*)) = 0$ at some finite n^*.

As δ increases, the equilibrium number of resource users in the commons falls. It is straightforward to show that for the parameter values described in Figure 4 if δ rises to 9.378, the equilibrium number of resource users falls to one — monopoly.

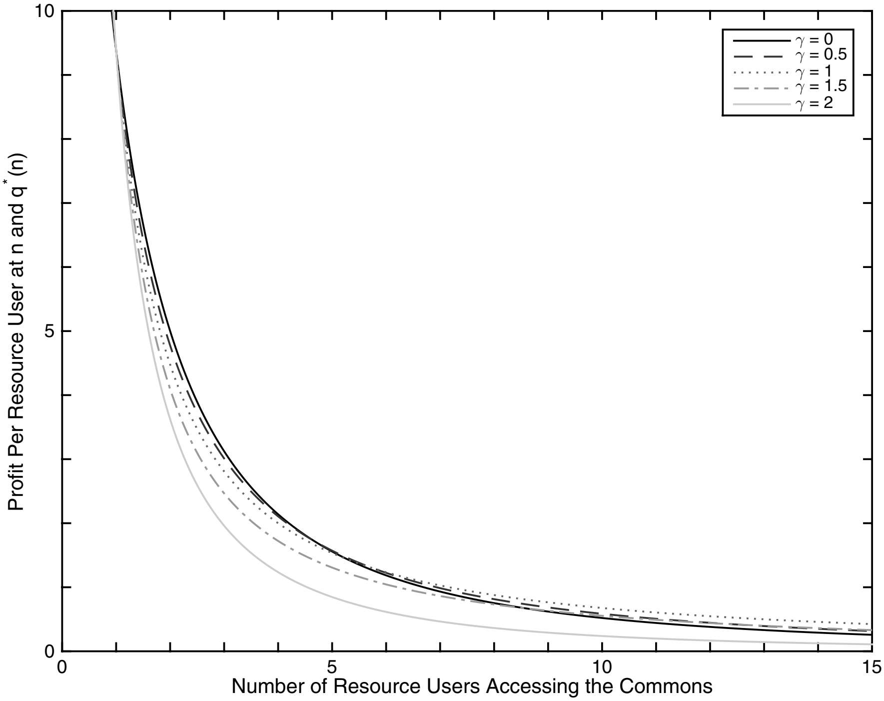

Figure 3: Profit per resource user for each n in the common when marginal cost is increasing in output and access cost is zero.
The curves show profits per resource user, π_i^b (Case 2 Equation (4)), as a function of the number of resource users in the commons, n, assuming they produce at $q^*(n)$ (Equation (8)) for various values of γ. Here, we assume no access cost ($\delta = 0$). Note that $q^*(n)$ is always positive; therefore, the equilibrium number of resource users is not limited by cost and is given by $n(0)$. Parameters: $l = 10$, $c = 1$.

4 Discussion and Concluding Remarks

In this paper, we have exploited the potential function in commons congestion games to identify the stochastically stable Nash equilibrium of the game. In so doing we have also verified our claim that when the cost function takes the form $cn^\gamma q_i$, (i.e., marginal cost is constant for a given n), we can identify the finite equilibrium number of resource users in the commons. However, if the cost function takes the form $cn^\gamma q_i^2$, the equilibrium number of resource users is the maximum number entitled to access the commons. This is because each resource user's profit is

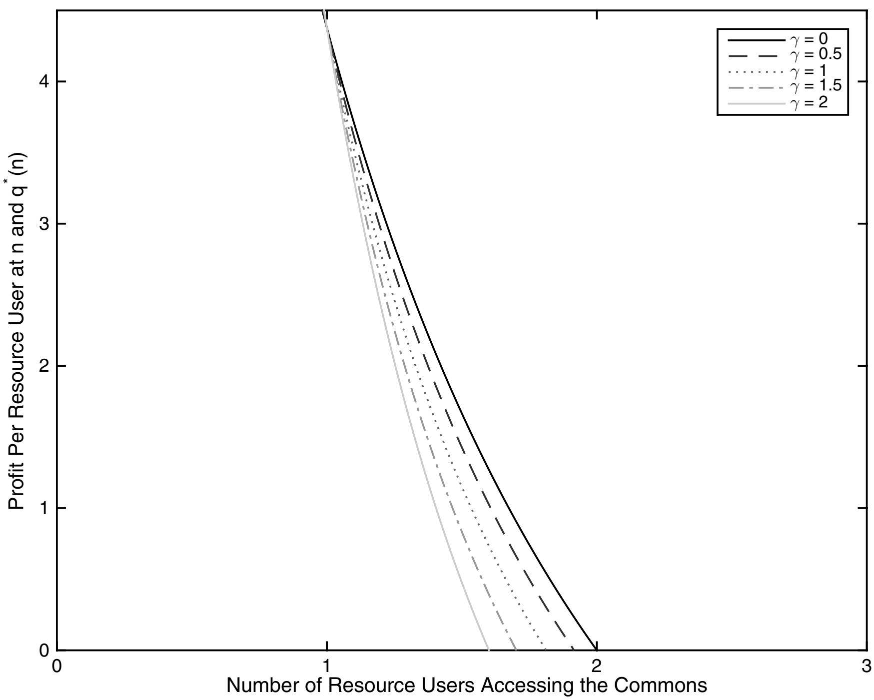

Figure 4: Profit per resource user for each n in the commons when marginal cost is increasing in output and access cost is positive.
The curves show profits per resource user, π_i^b (Case 2 Equation (4)), as a function of the number of resource users in the commons, n, assuming they produce at $q^*(n)$ (Equation (8)) for various values of γ. Here, we assume that access cost is positive ($\delta = 5$). The equilibrium number of resource users is given by $\pi_i^b(n^*, q^*(n^*)) = 0$, where the profit curves intersect with the horizontal axis. Specifically, note that n^* decreases as γ increases. Parameters: $l = 10$, $c = 1$.

positive for all $n \geq 0$. We find that the tragedy of the commons is the product of cost structures in which either:

(a) the cost of production is not increasing in the number of resource users or the level of output, or

(b) the cost of production is increasing in the number of resource users or the level of output but at a lower rate than the increase in revenue.

Our results provide an alternative to the coordination or cooperative game focus of Ostrom and colleagues, who are concerned with the establishment of rules that determine which members of society are entitled to access common pool resources, and under what conditions. If there is a relationship between the cost structures that determine the equilibrium number of resources users under open access and common pool resource protocols, however, the two approaches should give equivalent results.

We acknowledge that not all common pool resources can be analyzed in this way. In some cases the number of resource users is fixed by decree. An example of a commons to which access is independent of costs is licensed common pool fisheries in Japan (Government of Japan, 2016). Fishing licenses are issued by the governor of a prefecture. Applicants are divided into two groups. The first group comprises fishers who reside in the area and have historically engaged in the industry, while the second group comprises of fishers from elsewhere who plan to start fishing. The first group is automatically given entry, but members of the second group must wait until there is a vacancy. Other examples identified in the literature include common pool forests, grasslands, wetlands, water resources, and hunting areas (Berkes *et al.*, 1989; Feeny *et al.*, 1990; McWhinnie, 2009).

There are nevertheless many examples of common pool resources in which the number of potential users is not restricted, but the number of actual users is sensitive to costs of access or costs of production. The example of the Maine lobster fishery mentioned in the introduction to this paper is a case in point. We suggest that participation in the fishery could be analyzed directly through the institutionally driven changes in access and production costs.

Internationally, there have been several common pool resources that have been depleted because access and/or production costs fell as a result of either technological developments or government subsidies on effort or capital equipment (Allen and Keay, 2001; Finlayson and McCay, 1998). Examples include the exploitation of sea areas beyond national jurisdiction (Allen and Keay, 2001). Take the case of whales. All countries have open access to the High Seas, and many countries have actively hunted whales in the past. Several whale species were severely depleted in the nineteenth century. This includes a number of Baleen Whales, targeted for their blubber, such as the Bowhead, Grey, Humpback, and Right Whales. Amongst toothed whales, the Sperm

Whale, hunted for spermaceti until the discovery of kerosene in the 1840s, was similarly depleted. In the twentieth century the range of whales exploited widened, the number of firms accessing whale fisheries increased, and the rate at which whale populations were harvested rose dramatically. Several stocks were driven down to commercial extinction. Aside from the Baleen Whales targeted in the nineteenth century, stocks of Blue, Fin, Sei, and Beluga Whales all crashed. It is estimated that just under 3 million whales were harvested between 1900 and 1986 when the International Whaling Commission approved a moratorium (Rocha *et al.*, 2014).

The driver of changes in number of whaling firms, the species targeted, and the level of harvest was, in every case, a change in profitability caused by changes in the cost of access or production, or by changes in demand (Davis *et al.*, 2007). The rapid decrease of Bowhead Whale in Eastern Arctic between the late eighteenth and early nineteenth centuries, for example, was due to both the payment of 'revenue bounties' aimed at increasing the size of whaling vessels, and productivity improvements caused by changes in hull design that reduced the cost of whale hunting (Allen and Keay, 2001). In the twentieth century, the introduction of diesel engines, factory ships, and explosive harpoons was amongst the supply side drivers of the growth in the numbers of whalers, but profitability was also affected by demand-side factors. At the time when the moratorium was declared, whaling was a rapidly declining industry due to the combined effects of declining stocks (which increased production costs), the emergence of substitute products, rising incomes, and internationally increasing environmentalism. Regulation followed, rather than led, catch changes (Schneider and Pearce, 2004).

Other examples of cost-led declines in common pool marine resources include the Atlantic Cod fishery. Overexploitation of cod stocks was partly due to the fact that the predicted rate of growth of the stock was greatly overestimated (Hutchings and Myers, 1994), which induced a significant amount of industrial investment. But it was also due to the effect of government subsidies for new vessels or for upgrades to fishing capacity, particularly after 1985/1986 (Finlayson and McCay, 1998). Similarly, the overexploitation of the Atlantic and Mediterranean Bluefin Tuna was due to the effect on costs of an increase in the size and power of French seiners, the introduction of new and more effective positioning and prospecting equipment, and the introduction of new storage equipment from the late 1980s to mid-1990s (Fromentin and Ravier, 2005).

In terrestrial systems, there are many parallel examples of cost-driven changes in the rate at which common pool resources have been extracted. Reductions in the cost of disease and disease control, for example, significantly increased the rate at which African savannas were exploited (Giblin, 1990; Steverding, 2008). The role of subsidies in accelerating the depletion of common pool forest resources has long been recorded (Barbier and Burgess, 2001; Binswanger, 1991; Heath and Binswanger, 1996). Common pool freshwater resources have also been affected. To take just one instance, groundwater reserves are frequently available to anyone with the capacity to drill to the water table (the cost of access). The cost of production in such cases is simply the cost of pumping plus the cost of surface storage and distribution systems. A study of groundwater use by farmers in the Hamadan–Bahar plain in Iran, for example, argued that groundwater depletion in the area is due both to the fact that farmers are not required to pay for water, and to the existence of a range of subsidies for agricultural production. The net effect is a reduction in the cost of production that has led both to the sinking of new wells and an increase in the rate at which water is pumped from existing wells (Balali *et al.*, 2011).

It is important to underline the fact that resource users are behaving efficiently from a private perspective, even in the case where costs induce the tragedy of the commons. If common pool resources or the societies dependent on them have collapsed, we argue that it may be because either access or congestion costs were inconsistent with the sustainable use of the resources. We have made the point that the institutional arrangements for the management of common pool resources recorded in the literature are likely to have implications for the structure of both access and production costs. The Maine lobster fishery is an example. This is not to undermine research into the conditions in which those with access rights to common pool resource seek to avoid 'the price of anarchy'. But it does suggest that the mechanism at work may be the resulting costs. If access or production costs — however they are determined — are consistent with the sustainable use of common pool resources, then the myopic self-interested behavior of those with rights of access cannot threaten those resources.

Appendix A

We show the case of π_i^b and Π^b. The case of π_i^a and Π^a is similar. We first verify that Π^b is a potential by showing that $\partial\Pi^b/\partial q_i = \partial\pi_i^b/\partial q_i$.

$$\begin{aligned}
\frac{\partial \Pi^b}{\partial q_i} &= l - n^2 q^{*2} - 2nq^*\left(\sum_{m=1}^{n} q_m - nq^*\right) - 2nq^* q_i - 2cn^\gamma q_i \\
&\quad - 2nq^*(q_1 + q_2 + \cdots + q_{i-1} + q_{i+1} \cdots + q_n) \\
&\quad + 2nq^*(q_1 + \cdots + q_n) - 2nq^* q_i \\
&= l - n^2 q^{*2} - 2nq^*\left(\sum_{m=1}^{n} q_m - nq^*\right) - 2nq^* q_i - 2cn^\gamma q_i \\
&= \frac{\partial \pi_i^b}{\partial q_i}
\end{aligned}$$

Hence, Π^b is a potential function.

To find the Nash equilibrium for a fixed n, first we examine the first-order condition.

$$\begin{aligned}
\frac{\partial \Pi^b}{\partial q_i} &= l - n^2 q^{*2} - 2nq^*\left(\sum_{m=1}^{n} q_m - nq^*\right) - 2nq^* q_i - 2cn^\gamma q_i \\
&= 0
\end{aligned}$$

Since the game is symmetric, $q_1^* = \cdots = q_n^* = q^*$. Hence, the first-order condition becomes

$$\begin{aligned}
&l - n^2 q^{*2} - 2nq^*(nq^* - nq^*) - 2nq^{*2} - 2cn^\gamma q^* = 0 \\
&\Leftrightarrow (n^2 + 2n)q^{*2} + 2cn^\gamma q^* - l = 0
\end{aligned}$$

Consequently, the critical point is

$$q^* = \frac{\sqrt{c^2 n^{2\gamma} + l(n^2 + 2n)} - cn^\gamma}{n^2 + 2n}$$

Note that $\sqrt{c^2 n^{2\gamma} + l(n^2 + 2n)} - cn^\gamma > 0$ always holds.

Next, we verify the second-order condition. For $q_1 = q_2 = \cdots = q_n = q^*$ to be the argmax of the potential, the Hessian matrix has to be negative definite at $q_1 = q_2 = \cdots = q_n = q^*$. To find the Hessian matrix, we need the following derivatives:

$$\frac{\partial^2 \Pi^b}{\partial q_i^2} = -4nq^* - 2cn^\gamma$$

and

$$\frac{\partial \Pi^b}{\partial q_j \partial q_i} = -2nq^*$$

for $i \neq j$. Hence, the Hessian matrix is

$$H = \begin{pmatrix} -4nq^* - 2cn^\gamma & -2nq^* & \dots & -2nq^* \\ -2nq^* & -4nq^* - 2cn^\gamma & \dots & -2nq^* \\ \vdots & \vdots & \ddots & \vdots \\ -2nq^* & -2nq^* & \dots & -4nq^* - 2cn^\gamma \end{pmatrix}$$

The eigenvalues are $n-1$ multiplicities of $-2cn^\gamma - 2nq^*$ and $-2cn^\gamma - 2n(n+1)q^*$, and clearly, both of them are negative. Hence, H is negative definite so that Π attains a local maximum at q^*.

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